

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

Level - 1	Daily Tutorial Sheet 1
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Note: In questions involving log, it's important to remember that $\log(p(x))$ is well defined only for $x : p(x) \in (0, \infty)$ and $q(x) \in (0, 1) \cup (1, \infty)$.

1.(-5) $x^2 - 2x + 65 = (5 - x)^2 = 25 + x^2 = 10x$

$$\Rightarrow 8x = -40$$

$$x = -5$$

2.(5) $\frac{3}{2} + \log_4 \frac{x+3}{x-1} = 2 \Rightarrow \log_4 \frac{x+3}{x-1} = \frac{1}{2} \Rightarrow \frac{x+3}{x-1} = 2 \Rightarrow x+3 = 2x-2 \Rightarrow x=5$

3.(4) $\sqrt{x+5} + \sqrt{x} = 5$

$$\Rightarrow 2x+5+2\sqrt{x(x+5)} = 25 \Rightarrow 4(x(x+5)) = (20-2x)^2 \Rightarrow x^2+5x = 100+x^2-20x$$

$$\Rightarrow x = 4$$

Alternate solution: $\sqrt{x+5} + \sqrt{x} = 5 \dots (i)$

$$(\sqrt{x+5} + \sqrt{x})(\sqrt{x+5} - \sqrt{x}) = 5(\sqrt{x+5} - \sqrt{x})$$

$$\Rightarrow 5 = 5(\sqrt{x+5} - \sqrt{x}) \Rightarrow \sqrt{x+5} - \sqrt{x} = 1 \dots (ii)$$

From (i) and (ii), $\sqrt{x+5} = 3$ and $\sqrt{x} = 2$

$x = 4$ satisfies both equations.

4.(2) $3^x - 8 = 3^2 \cdot 3^{-x} \Rightarrow y - 8 = \frac{9}{y} \dots (y = 3^{+x}) \Rightarrow y^2 - 8y - 9 = 0 \Rightarrow (y+1)(y-9) = 0$

$$\Rightarrow y = -1, 9 \Rightarrow y = 9 \Rightarrow x = +2$$

5.(-2) $\log_2(25^{x+3} - 1) = \log_2 4(5^{x+3} + 1)$

Let $5^{x+3} = y$

$$\Rightarrow y^2 - 1 = 4y + 4 \Rightarrow y^2 - 4y - 5 = 0 \Rightarrow (y-5)(y+1) = 0$$

$$\Rightarrow y = -1, 5 \Rightarrow 5^{x+3} = 5 \dots [y = -1 \text{ not possible}] \therefore x = -2$$

6.(1) $\frac{4 \cdot 3^x - 6}{9^x - 6} = 2$

$$\Rightarrow 4y - 6 = 2y^2 - 12 \dots [y = 3^x] \Rightarrow 2y^2 - 4y - 6 = 0 \Rightarrow y^2 - 2y - 3 = 0$$

$$\Rightarrow (y-3)(y+1) = 0 \Rightarrow 3^x = 3 \Rightarrow x = 1$$

7.(2) $2 \times 4^{x-2} - 1 = 4^{2x-4}$

$$\Rightarrow \frac{2 \times y}{4^2} - 1 = \frac{y^2}{4^4} \dots [4^x = y] \Rightarrow (2y-16)4^2 = y^2 \Rightarrow 32y - 256 = y^2$$

$$\Rightarrow y^2 - 32y + 256 = 0 \Rightarrow (y-16)^2 = 0 \Rightarrow 4^x = y = 16 \therefore x = 2$$

$$\begin{aligned}
 \mathbf{8.(1/2, 1/4)} \quad 5^{1/x} + 125 &= 5^{\left(\log_5 6 + 1 + \frac{1}{2x}\right)} = 6 \times 5 \times 5^{\frac{1}{2x}} \\
 \Rightarrow y^2 + 125 &= 30y \cdot [y = 5^{1/2x}] \Rightarrow y^2 - 30y + 125 = 0 \Rightarrow (y - 25)(y - 5) = 0 \\
 \Rightarrow y &= 5, 5^2 \Rightarrow \frac{1}{2x} = 1, 2 \Rightarrow x = \frac{1}{2}, \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{9.(1)} \quad \log_{10} 10^x + \log_{10} (2^x + 1) &= \log_{10} 6 + \log_{10} 5^x \\
 \Rightarrow \log_{10} 10^x (2^x + 1) &= \log_{10} 6 \cdot 5^x \Rightarrow 10^x (2^x + 1) = 6 \cdot 5^x \Rightarrow 2^x (2^x + 1) = 6 \\
 \Rightarrow y^2 + y - 6 &= 0 \Rightarrow (y + 3)(y - 2) = 0 \Rightarrow y = 2, -3 \Rightarrow 2^x = 2, -3 \Rightarrow x = 1
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{10.(23, -9/5)} \quad 1 + \frac{2}{y} &= y \dots [y = \log_5 (x + 2)] \Rightarrow y^2 - y - 2 = 0 \Rightarrow (y - 2)(y + 1) = 0 \Rightarrow y = 2, -1 \\
 \Rightarrow x + 2 &= 5^2, 5^{-1} \Rightarrow x = 23, \frac{1}{5} - 2 = 23, -\frac{9}{5}
 \end{aligned}$$

$$\mathbf{11.(2 - \sqrt{2})}$$

Note: The given equation is well defined only for $1 - x \in (0, 1) \cup (1, \infty)$ and $3 - x \in (0, 1) \cup (1, \infty)$.
i.e. $x \in (-\infty, 1) - \{0\}$.

$$\begin{aligned}
 y = \frac{1}{y} \Rightarrow y^2 &= 1 \Rightarrow y = \pm 1 \Rightarrow \log_{1-x} (3 - x) = \pm 1 \Rightarrow 3 - x = (1 - x) \text{ or} \\
 3 - x &= \frac{1}{1 - x} \Rightarrow 3 = 1 \text{ or } x^2 - 4x + 3 = 1 \Rightarrow x^2 - 4x + 2 = 0 \\
 x &= \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2} \quad x = 2 + \sqrt{2} \text{ lies outside the domain of } x \text{ and hence } x = 2 - \sqrt{2} \text{ is the only valid} \\
 &\text{solution.}
 \end{aligned}$$

12.(1/4) Note: The given equation is well defined only for $1 - 2x \in (0, 1) \cup (1, \infty)$ and $1 - 3x \in (0, 1) \cup (1, \infty)$.
i.e. $x \in (-\infty, 1/3) - \{0\}$.

$$\begin{aligned}
 \log_{1-2x} (6x^2 - 3x - 2x + 1) - \log_{1-3x} (4x^2 - 4x + 1) &= 2 \\
 \Rightarrow \log_{(1-2x)} [(2x - 1)(3x - 1)] - \log_{(1-3x)} (2x - 1)^2 &= 2 \\
 \Rightarrow \log_{(1-2x)} [(1 - 2x)(1 - 3x)] - \log_{(1-3x)} (1 - 2x)^2 &= 2 \Rightarrow 1 + y - \frac{2}{y} = 2 \dots [y = \log_{(1-2x)} (1 - 3x)] \\
 \Rightarrow y = 2, -1 \Rightarrow (1 - 3x) &= (1 - 2x)^2 \text{ or } (1 - 3x) = \frac{1}{(1 - 2x)} \Rightarrow x = 0, \frac{1}{4} \text{ or } x = 0, \frac{5}{6}
 \end{aligned}$$

Out of these values of x only $x = \frac{1}{4}$ lies in the domain.

$$\begin{aligned}
 \mathbf{13.(100)} \quad 7^{\log x} &= 98 - x^{\log 7} \\
 \Rightarrow 7^{\log x} &= 98 - 7^{\log x} \Rightarrow 7^{\log x} = 7^2 \Rightarrow \log x = 2 \\
 x &= 100
 \end{aligned}$$

$$\mathbf{14.(25)} \quad \sqrt{1 + \frac{1}{2} \log_{0.2} x} + \sqrt{3 + \log_{0.2} x} = 1$$

$$\begin{aligned} \Rightarrow \sqrt{1+\frac{y}{2}} + \sqrt{3+y} &= 1.. \left[y = \log_{0.2} x \right] \Rightarrow 4 + \frac{3y}{2} + 2\sqrt{\left(1+\frac{y}{2}\right)(3+y)} = 1 \\ \Rightarrow 4\left(1+\frac{y}{2}\right)(3+y) &= \left(-3 - \frac{3y}{2}\right)^2 \text{ {squaring both sides}} \\ \Rightarrow 4\left(1+\frac{y}{2}\right)(3+y) &= \left(-3 - \frac{3y}{2}\right)^2 \\ \Rightarrow 4\left(3+y + \frac{3y}{2} + \frac{y^2}{2}\right) &= 9 + \frac{9y^2}{4} + 9y \Rightarrow \frac{y^2}{4} - y - 3 = 0 \Rightarrow y^2 - 4y - 12 = 0 \\ \Rightarrow (y-6)(y+2) &= 0 \\ \Rightarrow y = -2 \text{ {y = 6 won't satisfy the original equation}} \\ \Rightarrow \log_{0.2} x &= -2 \\ \Rightarrow x &= 25 \end{aligned}$$

Alternate solution:

$$\begin{aligned} \sqrt{1+\frac{y}{2}} + \sqrt{3+y} &= 1.. \left[y = \log_{0.2} x \right] \\ \Rightarrow \sqrt{3+y} &= 1 - \sqrt{1+y/2} \end{aligned}$$

Squaring both sides:

$$\begin{aligned} 3+y &= 1 + 1 + \frac{y}{2} - 2\sqrt{1+y/2} \\ \Rightarrow 1+y/2 &= -2\sqrt{1+y/2} \quad \dots (i) \end{aligned}$$

Note that: $\sqrt{1+y/2}$ is well defined only if $(1+y/2) \geq 0$. In the above equation,

LHS ≥ 0 and RHS ≤ 0

Thus, the only possible solution of the equation is LHS = RHS = 0

$$\Rightarrow 1+y/2 = 0$$

$$\Rightarrow y = -2$$

$$\Rightarrow x = (0.2)^{-2} = 25$$

15.(3) The given equation is well defined only for $x > 0$

$$4^{\log_2 2x} = 36 = 4^{\log_4 36} \Rightarrow \log_2 2x = \log_4 36 = \frac{1}{2} \log_2 36 \Rightarrow \log_2 (2x)^2 = \log_2 36$$

$$\Rightarrow (2x)^2 = 36$$

$$\Rightarrow 2x = 6 \dots [\text{as } x > 0] \Rightarrow x = 3$$